Paths, Pivots and Practice: The Power of Optimization

Conference Program

June 11, 2015

08:15 Breakfast (provided)

08:45 Opening remarks

09:00 Conic optimization goes classic in many ways

Bomze, Immanuel, University of Vienna, immanuel.bomze@univie.ac.at

The fruitful approach to optimize over intractable convex cones has shattered Rockafellar's misleading adage of convexity as a watershed of hardness. Fifteen years after Tamás Terlaky, together with the Delft group (and some help from Vienna) coined the term Copositive Optimization, we witness a transition phase, a development Science and (Applied) Art share - from invention (and originality) to artisan craftsmanship profiting from innovation and investing creativity into diverse applications, extending the effective radius of creation. The adjective "classic" reflects my view that nowadays this approach should be taught as a basic tool very much like LP and IPM theory.

09:30 Facial reduction for cone optimization

Wolkowicz, Henry, University of Waterloo, hwolkowicz@uwaterloo.ca

The Slater constraint qualification (SCQ) is essential for many classes of convex programs, e.g., Linear Programming (LP), ordinary convex programming (CP), and cone optimization (CO). However, SCQ fails for many problems, e.g., for many instances of semidefinite programming (SDP) that arise from relaxations of computationally hard problems. This degeneracy results in theoretical problems (possible loss of strong duality) as well as numerical problems (due to ill-posedness). A theoretical tool to regularize these problems uses facial reduction. We present a backwards stable approach for preprocessing a general SDP using facial reduction. In addition, we consider several specific applications where the structure of the problem surprisingly allows us to exploit the degeneracy.

10:00 Exact duality in semidefinite programming based on elementary reformulations

Pataki, Gabor, UNC Chapel Hill, gabor@unc.edu

In semidefinite programming (SDP), unlike in linear programming, Farkas' lemma may fail to prove infeasibility. Here we obtain an exact, short certificate of infeasibility in SDP by an elementary approach: we reformulate equality constrained semidefinite systems using only elementary row operations, and rotations. When a system is infeasible, the reformulated system is trivially infeasible. When a system is feasible, the reformulated system has strong duality with its Lagrange dual for all objective functions. As a corollary, we obtain algorithms to generate the data of all infeasible SDPs and the data of all feasible SDPs whose maximum rank solution has a prescribed rank.

10:30 Coffee break (provided)

11:00 Computing lower and upper bounds on the extension complexity of polytopes

Glineur, Francois, Université catholique de Louvain, francois.glineur@uclouvain.be

A polytope can often be expressed as the projection of a higher-dimensional polytope, leading to a so-called extended formulation over which one can optimize more efficiently. We want to find extended formulations with minimum number of facets. This number, called the extension complexity, is equal to the nonnegative rank of the slack matrix of the polytope, and is NP-hard to compute. We will first survey lower bounds on this quantity. Then, we will show how local optimization algorithms can be used as heuristics to provide good upper bounds, and report on their efficiency on a set of concrete polytopes.

11:30 Edges vs circuits: A hierarchy of diameters in polyhedra

Borgwardt, Steffen, University of California Davis, sborgwardt@math.ucdavis.edu

The study of the combinatorial diameter of polyhedra is a classical open problem in the theory of linear optimization. We introduce a hierarchy of so-called circuit diameters, generalizations to the notion of combinatorial diameter, which in particular provide lower bounds on the usual diameter. After turning to the structure of the hierarchy and discussing these bounds, we explain the differences and similarities among the different diameter notions. We conclude with a discussion of some classes of polyhedra to highlight the insight gained from these studies.

12:00 New bounds for the max-k-cut and chromatic number of a graph

Sotirov, Renata, Tilburg University, r.sotirov@uvt.nl

In this talk we consider several SDP relaxations for the max-k-cut problem. The optimal solution of the weakest presented SDP relaxation has a closed form expression that includes the largest Laplacian eigenvalue of the graph under consideration. This bound is exploited to derive an eigenvalue bound on the chromatic number of a graph. For regular graphs, the new bound on the chromatic number is the same as the well-known Hoffman bound. We also demonstrate the quality of the presented bounds for several families of graphs, such as walk-regular graphs and graphs from the Hamming association scheme.

12:30 Lunch (on your own)

14:00 Perspectives and extensions of Renegar’s efficient first-order methods for conic optimization

Freund, Robert, MIT, rfreund@mit.edu

In 2014 Renegar presented a novel nonlinear transformation of a conic optimization problem into a non-smooth convex optimization problem over a simple set, which is ideally-suited for solution by first-order methods. He further improved the complexity of his method from a quadratic to a logarithmic dependence on the quality of the initial point through a novel “sub-scheme.” In this talk we present a new perspective on Renegar’s work that places it in the context of projective transformations and projective geometry and that mitigates the need for any sub-scheme to achieve improved complexity. Our results extend to improved complexity of first-order methods for a particular structured class of convex optimization problems.

14:30 The simplex method and strongly polynomial algorithms

Mizuno, Shinji, Tokyo Institute of Technology, mizuno.s.ab@m.titech.ac.jp

In this talk, we present an algorithm for LP, which uses Tardos' basic algorithm and solves auxiliary LP problems by the simplex method with Dantzig's rule. It is shown that the total number of distinct basic solutions generated by the proposed algorithm is bounded by a polynomial function of the number of constraints, the number of variables, and the maximum determinant of submatrices of a coefficient matrix. If the coefficient matrix is totally unimodular and all the auxiliary problems are nondegenerate, then the algorithm is strongly polynomial. We present both primal and dual versions of the algorithm.

15:00 On the turing model complexity of interior point methods for SDP

De Klerk, Etienne, Tilburg University, e.deklerk@uvt.nl

Semidefinite programming (SDP) is used in many polynomial-time approximation algorithms, like the Goemans-Williamson algorithm for the maximum cut problem. To give a rigorous proof of the polynomial running time, the ellipsoid method of Yudin and Nemirovski is usually invoked, since the Turing model complexity of the more practical interior point methods (IPMs) is not well-understood. In this talk we show how one may obtain rigorous complexity results for IPMs in the SDP case.

15:30 Coffee break (provided)

16:00 Unconstrained trust region based stochastic optimization with biased and unbiased noise.

Scheinberg, Katya, Lehigh University, katyas@lehigh.edu

We will present a very general framework for unconstrained stochastic optimization which is based on standard trust region framework using random models. In particular this framework retains the desirable features such step acceptance criterion, trust region adjustment and ability to utilize of second order models. We make assumptions on the stochasticity that are different from the typical assumptions of stochastic and simulation-based optimization. In particular we assume that our models and function values satisfy some good quality conditions with some probability fixed, but can be arbitrarily bad otherwise. We will analyze the convergence of this general framework and discuss the requirement on the models and function values. We will will contrast our results with existing results from stochastic approximation literature. We will then present computational results for several classes of noisy functions, including cases when noise is not i.i.d. and dominates the function values, when it occurs. We will show that our simple framework performs very well in that setting, while standard stochastic methods fail.

16:30 Chance constrained optimization for pari-mutuel horse race betting

Metel, Michael, McMaster University, metelm@mcmaster.ca

We consider the time horizon of a gambler in the optimization of horse race betting through the use of chance constrained programming. A novel approach to estimating superfecta payouts is presented using maximum likelihood estimation. A computational substantiation with historical race data found an increase in return of over 10% using the chance constrained model.

17:00 Solving generalized Nash games with evolutionary variational inequalities

Cojocaru, Monica-Gabriela, University of Guelph, mcojocar@uoguelph.ca

We show in this talk how a new parametrization technique can be introduced (via evolutionary variational inequality (EVI) problems) such that by restricting the solution sets of such specialized EVI problems with complementarity conditions, we obtain a description of the solution set of a generalized Nash (GN) game with shared constraints. We solve known examples and show that this approach solves GN previously not solved by existing VI parametrization techniques. We apply the method to several examples, including a vaccinating game in a heterogeneous population where policy decisions are modelled as shared constraints.

June 12, 2015

08:30 Breakfast (provided)

09:00 Paths, pivots, and fixed-charge capacitated multi-commodity network design

Gendreau, Michel, Polytechnique Montréal, michel.gendreau@cirrelt.ca

We survey more than 15 years of work on the development of effective metaheuristics for tackling the Fixed-Charge Capacitated Multi-commodity Network Design Problem, which is a problem of great practical importance in transportation planning. Interestingly enough, this line of research involved manipulating paths through some pivot operations. Cycles and Path Relinking were also part of the story...

09:30 Vector space decomposition for network and linear programs

Desrosiers, Jacques, HEC Montréal, jacques.desrosiers@hec.ca

VSD is a generic primal algorithmic framework. It moves from a solution to the next according to a direction and a step size. The core component is obtained via the computation of the smallest reduced cost upon dividing the set of dual variables: one is fixed whiles the other is optimized. Special cases are the Primal Simplex, the Minimum Mean Cycle-Canceling algorithm for networks, the Dynamic Constraint Aggregation for Set Partitioning and the Improved Primal Simplex. Properties of VSD allow identifying cases that avoid degenerate pivots and find interior directions.

10:00 Integral simplex using decomposition with primal cutting planes

Rosat, Samuel, GERAD, samuel.rosat@gerad.ca

The Integral Simplex Using Decomposition of Zaghrouti et al. (2014) is an augmenting method for the Set Partitioning Problem (SPP), based on all-integer simplex pivots. Given an integral solution of SPP, it solves a subproblem to determine a set of nonbasic columns to be pivoted in the basis to ensure a strict improvement of the solution, while the new solution is guaranteed to be integer. We show that cutting-planes can be transferred to the subproblem, and characterize the set of transferable cuts as a nonempty subset of primal cuts that are tight for the current solution. We propose efficient separation procedures for primal clique and odd-cycle cuts, and prove that their search space can be restricted to a small subset of the variables. A geometrical interpretation of these cutting-planes is given. Numerical results demonstrate the efficiency of adding cutting-planes to the algorithm; tests are performed on small and large-scale set-partitioning problems from aircrew and bus-driver scheduling instances up to 1,600 constraints and 570,000 variables.

10:30 Coffee break (provided)

11:00 An LP-based algorithm to test copositivity

Yoshise, Akiko, University of Tsukuba, yoshise@sk.tsukuba.ac.jp

In this talk we propose a new branch and bound type algorithm for testing copositivity. Two features of our algorithm are: (1) we introduce new classes of matrices which are relatively large subsets of the set of copositive matrices and work well to check copositivity, and (2) for incorporating the sets in checking copositivity, we only have to solve a linear optimization problem after computing a singular value matrix decomposition, which implies that our algorithm is not so time-consuming. Our preliminary numerical experiments suggest that our algorithm is promising for determining upper bounds of the maximum clique problem.

11:30 Completely positive relaxations of quadratically constrained quadratic optimization

Zuluaga, Luis, Lehigh University, lzuluagag@gmail.com

There is a well-established body of research on quadratic optimization problems based on reformulations of the original problem as a conic optimization problem over the cone of completely positive matrices, or its conic dual, the cone of copositive matrices. As a result of this reformulation approach, novel solution schemes for quadratic polynomial optimization problems have been designed by drawing on conic optimization tools, and the extensively studied cones of completely positive and of copositive matrices. In particular, this approach has been applied to address key combinatorial optimization problems. Along this line of research, we consider quadratically constrained quadratic optimization problems and provide sufficient and necessary conditions for this type of problems to be reformulated as a conic optimization problem over the cone of completely positive matrices. Thus, recent related results for quadratic problems can be further strengthened. Moreover, these results can be generalized to optimization problems involving higher order polynomias.

12:00 Completely positive reformulations for optimization problems with complementarity constraints

Mitchell, John, RPI, mitchj@rpi.edu

We derive equivalent convex completely positive reformulations for several classes of nonconvex optimization problems defined over convex cones, including rank-constrained semidefinite programs and quadratically constrained quadratic programs (QCQPs). The first part of the reformulation is to cast the problem as a conic QCQP with just one nonconvex constraint with a special structure. Our results do not make any boundedness assumptions on the feasible regions of the problems considered.

12:30 Lunch (on your own)

14:00 Second-order cone programming for nonnegative regression with P-Spline

Xia, Yu, Lakehead University, yxia@lakeheadu.ca

We consider regression by B-splines with a penalty on high-order finite differences of the coefficients of adjacent B-splines. The penalty prevents overfitting. The underlying function is assumed to be nonnegative. The model is casted as a second-order cone programming problem, which can be solved efficiently by modern optimization techniques. The method is implemented in MATLAB.

14:30 Some ideas about branch and bound approach to SOCP problems with mixed integer constraints

Alizadeh, Farid, Rutgers University, alizadeh@rci.rutgers.edu

With the aid of clever cutting planes, branch and bound methods for integer programs with linear objective and constraints have been used successfully for decades to solve very large problems. More recently integer and binary constraints have been arisen in convex conic optimization problems more general than linear programs. One of the major computational impediments for duplicating branch and bound techniques for such problems effectively is the lack of simplex algorithms. While dual simplex method has been used quite effectively for linear integer programs there is no counterpart for more general conic problems that can take advantage of "warm-start's". In this talk we review some specific second order cone problems with combinatorial/integer constraints. Among these are the k-min-ball problem (find the smallest ball containing at least k of n points in d-dimensional Euclidean space), the Euclidean Steiner tree problem, the k- Fermat-Weber problem (find a point x and a subset of at least k points out of given n points in Euclidean d-space such that the sum of distances of these k points to x is minimal), The k-Markowitz portfolio optimization problem (from a set of n assets with known expected return and covariance matrix, find a subset of at most k assets minimizing risk while achieving at least an expected return of R). All such problems are SOCP (and sometimes QP) problems with binary constraints. We discuss how extensions of the simplex method and other first order methods may be incorporated in a branch and bound scheme to solve them effectively.

15:00 Structural analysis of ill-conditioned semidefinite/second-order cone programs

Tsuchiya, Takashi, National Graduate Institute for Policy Studies, tsuchiya@grips.ac.jp

We present a structural analysis of semidefinite feasibility problems and second-order cone feasibility problems. This is done by decomposing the problem into smaller problems, in a way that most feasibility properties of the original problem are preserved. With this technique, we understand clearly and systematically how weak infeasibility arises in SDP and SOCP. We also show that for a weakly infeasible problem over n by n matrices or the direct product of n second-order cones, at most n-1 directions are required to approach the cone. Some issues on feasibility certificates and computational complexity are also discussed.

15:30 Coffee break (provided)

16:00 Disjunctive conic cuts for mixed integer second order cone optimization

Goez, Julio, Polytechnique Montréal, jgoez@hotmail.com

We provide an overview about disjunctive conic cuts for mixed integer second order cone optimization (MISOCO). A MISOCO problem minimizes an affine function over the intersection of an affine subspace with the Cartesian product of second order cones. Additionally, a subset of the decision variables are constrained to take integer values. We extend the ideas of disjunctive programming, which have been very effective in the derivation of linear cuts for mixed integer linear optimization. Applying this extension to MISOCO problems results in the derivation of some novel conic cuts.

16:30 New global algorithms for linearly constrained quadratic programming

Peng, Jiming, University of Houston, jopeng@uh.edu

LCQP has been recognized to be among the hardest optimization problems, and even locating a local minimum to it is NP-hard. In this talk, we introduce a new class of algorithms for LCQP based on several simple effective optimization techniques such as alternative update, convex relaxation, initialization and partitioning. Global convergence and complexity will be discussed.

17:00 Pivots for Fitness

Fukuda, Komei, ETH Zurich, fukuda@math.ethz.ch

The beauty of the Criss-Cross pivoting method by Terlaky and Wang is its simplicity and strength in solving any general LP in finite steps. In fact it only needs the sign information of LP dictionaries and can be used in a purely combinatorial and topological setting of oriented matroids. In this talk, we discuss how to remove all redundant inequalities from a given system of linear inequalities by using only the signs of LP dictionaries. The problem of removing redundancies is obviously polynomially-solvable by many linear programs, but yet it is very time-consuming for large linear systems. It is a little like removing excess weights from our bodies (especially after 60 years old). It is always possible with exercises and restraints, but it is much harder to do than to talk about. Here our main goal is to find a simple and most practical pivot algorithm for redundancy detection and removal.

17:30 Special session followed by Banquet (provided)

June 13, 2015

08:30 Breakfast (provided)

09:00 Robotic path-finding techniques in stereotactic radiosurgery treatment optimization

Aleman, Dionne M., University of Toronto, aleman@mie.utoronto.ca

We investigate applying robotic path finding techniques to develop treatment plans for Gamma Knife Perfexion where the radiation is delivered continuously. We explore the use of simultaneous localization and mapping, combined with heuristic exploration techniques, to generate a path. A mixed integer model is then used to find the beam times for this selected path. We discuss the advantages and challenges of this method in comparison to the conventional forward and inverse step-and-shoot plans.

09:30 A multi-haul model for vertical alignment optimization of road design

Hare, Warren, The University of British Columbia, Warren.Hare@ubc.ca

The vertical alignment optimization problem for road design aims to generate a vertical alignment of a new road with a minimum cost, while satisfying safety and design constraints. We present a new model called multi-haul quasi network flow (MH-QNF) for vertical alignment optimization that improves the accuracy and reliability of previous mixed integer linear programming models.

10:00 Industrial optimization with big data: Opportunities and challenges

Hassini, Elkafi, McMaster University, hassini@mcmaster.ca

The presenter will share his recent experience on working on industrial optimization projects that involve big data as well setting up an advanced business data analytics research lab that is partly funded by Canada Foundation for Innovation (CFI). In addition, there will be a discussion on current work on a big data optimization tool, DooPlex, that facilitates the implementation of Cplex on Hadoop. The talk will conclude with some thoughts on future directions, including optimization with quantum computing.

10:30 Coffee break (provided)

11:00 Active set methods for convex quadratic problems with simple bounds

Rendl, Franz, Alpen-Adria University Klagenfurt, rendl@aau.at

Hintermueller, Ito and Kunisch (SIOPT 2002) introduce a semismooth Newton method to solve convex quadratic problems with simple bound constraints. They show a very impressive computational behaviour of their method. The method is known to converge only under some form of diagonal dominance of the Hessian or if the Hessian belongs to some special class of matrices, in general however it may cycle. We propose modifications of this method and show global convergence for positive definite Hessians. The computational performance of the new methods is only slightly higher than the original semismooth Newton method, but convergence is guaranteed even for badly conditioned positive definite Hessians.

11:30 Globally optimized packings of non-uniform size circles and spheres: A new algorithmic solution approach, with illustrative results

Pinter, Janos D., PCS Inc, janos.d.pinter@gmail.com

The general circle (sphere) packing problem considered here is to find a globally optimized, pairwise non-overlapping arrangement of N arbitrary sized circles (spheres) inside a container circle (sphere) with minimal radius. We present corresponding scalable optimization models, and then solve these using our MathOptimizer Professional (MOP) software. MOP is the LGO solver suite for global and local nonlinear optimization, linked to the integrated computing system Mathematica. The generic solver approach implemented in LGO is enhanced by customized heuristics. Illustrative numerical results are presented for configurations with up to 50 circles and spheres.

12:00 Closing remarks. End of conference.